EFFECT OF THE PARAMETERS OF ORTHOTROPY ON THE SPECTRUM IN PROBLEMS OF VIBRATIONS OF SHELLS

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The need for investigation of the distribution of the natural vibrations of thin elastic shells developed after the appearance of [1], in which it was remarked that the spectrum in problems of vibrations has points of bunching, whose existence is due to the specific character of the equations. Review [2] sets forth the present state of the theory of the distribution of frequencies. The same kind of approach was used in an investigation of the spectra of vibrations of orthotropic round cylindrical shells [3]. The hyperbolic-type integrals obtained were calculated in a computer. In [4] an investigation was made of the vibrations of flat orthotropic shells of arbitrary curvature; the hyperbolic-type integral is reduced to an analytical integral by the introduction of an approximate relationship. In connection with mechanical applications, the factors of most interest are the initial points and the bunching points of the spectrum, as well as their mutual arrangement depending on the geometry of the properties of the materials. We consider below the asymptotic function of the distribution, the asymptotic density of the natural frequencies, and the bunching points of the spectrum.

1. The equations of the free vibrations of flat orthotropic shells are written in the form [5]

$$L_1(c_{jk})w + \Delta_k\psi = \rho h\Omega^2 w, \quad L_2(c_{jk})\psi - \Delta_k w = 0, \tag{1.1}$$

$$L_1(c_{jk}) = \frac{h^3}{12}\left[c_{11}\frac{\partial^4}{\partial x^4} + 2(c_{12}+2c_{33})\frac{\partial^4}{\partial x^2\partial y^2} + c_{22}\frac{\partial^4}{\partial y^4}\right],$$

$$L_2(c_{jk}) = \frac{1}{h\left(c_{11}c_{22}-c_{12}^2\right)}\left[c_{11}\frac{\partial^4}{\partial x^4} + \frac{c_{11}c_{22}-c_{12}^2-2c_{12}c_{33}}{c_{33}}\frac{\partial^4}{\partial x^2\partial y^2} + c_{22}\frac{\partial^4}{\partial y^4}\right],$$

$$\Delta_k = \frac{1}{R_2}\frac{\partial^2}{\partial x^2} + \frac{1}{R_1}\frac{\partial^2}{\partial y^2},$$

where $c_{jk}(j, k = 1, 2)$, c_{33} are the elastic constants of the material, R_1, R_2 are the radii of curvature of the shell.

The asymptotic formula for the frequencies of the vibrations of shells, rectangular in a plan view, has the form [6]

$$\Omega^2R^4\rho h^{-2} = c_{11}k_m^4 + 2(c_{12}+2c_{33})k_m^2k_n^2 + c_{22}k_n^4 + \varkappa^4\left(k_m+\chi k_n^2\right)^2\Big/\left[c_{11}k_m^4 + \frac{c_{11}c_{22}-c_{12}^2+2c_{12}c_{33}}{c_{33}}k_m^2k_n^2 + c_{22}k_n^4\right], \tag{1.2}$$

$$k_m = m\pi R/a, \quad k_n = n\pi R/b, \quad \varkappa^4 = \frac{12R^4\left(c_{11}c_{22}-c_{12}^2\right)}{R_2^2h^2}, \quad \chi = R_2/R_1,$$

where a, b are the dimensions of the shell in a plan view, $|\chi| \leqslant 1$, since the coordinate axes can always change places. The applicability of formula (1.2) for determination of the frequencies of vibrations was considered in [6], where the conditions for the degeneration of the dynamic edge effect are determined. For freely supported shells, formula (1.2) is exact.

In the plane of the wave numbers k_m, k_n we introduce the system of coordinates

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$$k_m = r\cos\theta,\ k_n = r\sin\theta\ (r \geqslant 0,\ 0 \leqslant \theta \leqslant \pi/2). \tag{1.3}$$

We substitute the expressions for k_m, k_n into (1.2). From the relationship obtained we obtain a formula for r, which, after still another substitution $\xi = \sin^2\theta$ $(0 \leqslant \xi \leqslant 1)$, assumes the form

$$r = \{\varkappa^2 \sqrt{\omega^2 W_2 - [1 - \xi(1-\chi)]^2}/(W_1 W_2)\}^{1/2}, \tag{1.4}$$

$$W_1 = c_{11}(1-\xi)^2 + 2(c_{12} + 2c_{33})\xi(1-\xi) + c_{22}\xi^2, \quad \omega = R^4\rho h^{-2}\Omega^2/\varkappa^2,$$

$$W_2 = c_{11}(1-\xi)^2 + \frac{c_{11}c_{22} - c_{12}^2 - 2c_{12}c_{33}}{c_{33}}\xi(1-\xi) + c_{22}\xi^2.$$

In the plane $k_m k_n$, the given formula determines the region of wave numbers $k_m k_n$, corresponding to identical values of ω. The sense of the value of r is such that the expression under the radical sign in the shaped brackets in (1.4) must be nonnegative, if only for one value of ω from the integral [0, 1] . The minimal value of ξ with which this condition is satisfied gives us the initial point of the spectrum of the problem, i.e., the question of determining the start of the spectrum comes down to investigation of the function

$$f(\omega, \xi) = \omega^2 W_2 - [1 - \xi(1-\chi)]^2,$$

having a second order with respect to ξ and depending in a complex manner on the parameters of orthotropy c_{jk}, c_{33} and the parameter χ, characterizing the geometry of the shell.

2. The number of natural vibrations (less than the given value of ω_0) is defined, following R. Courant (see [2]), as the ratio of the area S in the plane of the wave numbers $k_m k_n$, inside of which $\omega \leqslant \omega_0$, to the area of one cell $\Delta k_m \Delta k_n$

$$N(\omega) = \frac{1}{\Delta k_m \Delta k_n} \iint\limits_S dk_m dk_n. \tag{2.1}$$

After the introduction of a new system of coordinates for k_m, k_n (1.3), we integrate expression (2.1) with respect to r

$$N(\omega) = \frac{ab}{2\pi^2 R^2} \int\limits_{\theta_1(\omega)}^{\theta_2(\omega)} r^2 d\theta. \tag{2.2}$$

We find the asymmetric density of the natural frequencies, differentiating (2.2) with respect to ω

$$M(\omega) = \frac{dN(\omega)}{d\omega} = \frac{ab}{2\pi^2 R^2}\left\{\int\limits_{\theta_1(\omega)}^{\theta_2(\omega)} \frac{dr^2(\omega,\theta)}{d\omega} d\theta - r^2(\omega,\theta_1)\frac{d\theta_1}{d\omega} + r^2(\omega,\theta_2)\frac{d\theta_2}{d\omega}\right\}. \tag{2.3}$$

We substitute into this the expression for r from (1.4). The terms outside the integral sign here are equal to zero. The formula (2.3) assumes the form

$$M(\omega) = \frac{ab}{2\pi^2 R^2} I, \quad I = \int\limits_{\alpha_1(\omega)}^{\alpha_2(\omega)} \omega \left[\frac{W_2}{\xi(1-\xi) f(\omega,\xi)}\right]^{1/2} d\xi. \tag{2.4}$$

The integral limits were determined from the conditions of the nonnegative character of the expressions under the radical sign in I, which is equivalent to satisfaction of the relationship $f(\omega, \xi) \geqslant 0$. The functions W_1, W_2 for all the parameters of the orthotropy with $0 \leqslant \xi \leqslant 1$ are positive. The bunching points of the spectrum are determined from the condi-

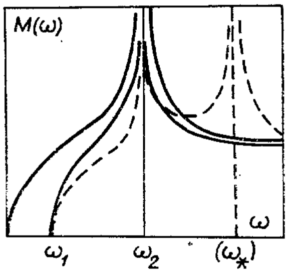

Fig. 1

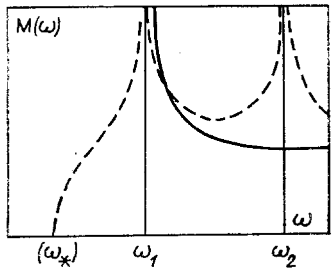

Fig. 2

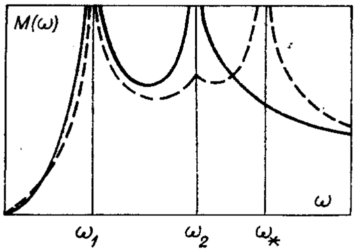

Fig. 3

tion of the divergence of integral (2.4). The necessary requirement for this is the equality to zero of the denominator in I

$$f_1(\omega, \xi) = \xi(1-\xi)\{\omega^2 W_2 - [1-\xi(1-\chi)]^2\} = 0. \tag{2.5}$$

In [4], the expression for the integration limit of (1.4) was replaced by an approximate expression, thanks to which the integral I can be expressed in terms of total elliptical integrals of the first kind in the Legendre form.

3. For the parameters of the orthotropy entering into (2.5) through W_2, the following possible relationships can be represented:

if $c_{22} > c_{11}$, then

$$c_{33} \leqslant \overset{*}{c}_{33}, \quad \overset{*}{c}_{33} \leqslant c_{33} \leqslant \overset{**}{c}_{33}, \quad c_{33} \geqslant \overset{**}{c}_{33}; \tag{3.1}$$

if $c_{11} > c_{22}$, then

$$c_{33} \leqslant \overset{**}{c}_{33}, \quad \overset{**}{c}_{33} \leqslant c_{33} \leqslant \overset{*}{c}_{33}, \quad c_{33} \geqslant \overset{*}{c}_{33}, \tag{3.2}$$

where

$$\overset{*}{c}_{33} = \frac{c_{11}c_{22} - c_{12}^2}{2(c_{22}+c_{12})}; \quad \overset{**}{c}_{33} = \frac{c_{11}c_{22} - c_{12}^2}{2(c_{11}+c_{12})}.$$

Let the parameters of the orthotropy satisfy the first relationship from (3.1). The curvature of the shell χ has a significant effect on the spectrum of the problem. Depending on the curvature of the shell there are three possible different cases.

1. In the first case

$$\chi \geqslant \frac{2c_{22}c_{33}}{c_{11}c_{22} - c_{12}^2 - 2c_{12}c_{33}} = \chi_1. \tag{3.3}$$

The expression in the right-hand part is greater than zero, but less than or equal to 1. The condition $f(\omega, \xi) \geqslant 0$, from which we seek the origin of the spectrum, starts to be satisfied with the frequency $\omega = \omega_*$, where

$$\omega_* = \left\{\frac{4c_{33}\left[c_{11}c_{33}\chi^2 - \left(c_{11}c_{22} - c_{12}^2 - 2c_{12}c_{33}\right)\chi + c_{11}c_{33}\right]}{\left(c_{11}c_{22} - c_{12}^2\right)\left[(c_{12}+2c_{33})^2 - c_{11}c_{22}\right]}\right\}^{1/2}. \tag{3.4}$$

With a given value of ω, the equation $f(\omega, \xi) = 0$ has a multiple root $\xi_1 = \xi_2$ in the interval of changes $0 \leqslant \xi \leqslant 1$. At all remaining points $\xi \in [0, 1]$, the function $f(\omega, \xi) < 0$. With calculation of r using formula (1.4), as well as with determination of the density of the natural frequencies using formula (2.4), we need the limits of change in the variable ξ, i.e., its values for which $f(\omega, \xi) \leqslant 0$. We denote them by $\alpha_1(\omega)$ and $\alpha_2(\omega)$. We also introduce the notation $\omega_1 = |\chi|/c_{22}^{1/2}, \quad \omega_2 = 1/c_{11}^{1/2}$.

The limits of change in the variable ξ are the following:

$$
\begin{gathered}
\omega \leqslant \omega_*, \; M(\omega) = 0, \\
\omega_* \leqslant \omega \leqslant \omega_1, \; \alpha_1(\omega) = \xi_1, \; \alpha_2(\omega) = \xi_2, \\
\omega_1 \leqslant \omega \leqslant \omega_2, \; \alpha_1(\omega) = 0, \; \alpha_2(\omega) = \xi_2, \\
\omega \geqslant \omega_2, \; \alpha_1(\omega) = 0, \; \alpha_2(\omega) = 1.
\end{gathered}
$$

To determine the bunching points of the spectrum we use the Cauchy criterion for the convergence of improper integrals. The integral I diverges with the following values of the parameter: $\omega = \omega_1$, $\omega = \omega_2$. With these values of ω, Eq. (2.5) has a multiple root at 0 or at 1, and these values of the variable ξ are the integration limits. If, in the first relationship of (3.1), the equality is satisfied, then, the expression in the right-hand part of (3.3) is equal to 1, if the equality is satisfied in (3.3), then $\omega_* = \omega_1$.

2. Let now the curvature of the shell satisfy the relationship

$$
0 \leqslant \chi \leqslant \frac{2c_{22}c_{33}}{c_{11}c_{22} - c_{12}^2 - 2c_{12}c_{33}}.
$$

The spectrum starts from the point $\omega = \omega_1$

$$
\begin{gathered}
\omega \leqslant \omega_1, \; M(\omega) = 0, \\
\omega_1 \leqslant \omega \leqslant \omega_2, \; \alpha_1(\omega) = 0, \; \alpha_2(\omega) = \xi_1, \\
\omega \geqslant \omega_2, \; \alpha_1(\omega) = 0, \; \alpha_2(\omega) = 1.
\end{gathered}
$$

The bunching point of the spectrum with $\omega = \omega_2$.

3. For shells with a negative Gaussian curvature ($\chi < 0$), the start of the spectrum at the point $\omega = 0$. ξ is varied within the limits

$$
\begin{gathered}
0 \leqslant \omega \leqslant \omega_1, \; \alpha_1(\omega) = \xi_1, \; \alpha_2(\omega) = \xi_2, \\
\omega_1 \leqslant \omega \leqslant \omega_2, \; \alpha_1(\omega) = 0, \; \alpha_2(\omega) = \xi_2, \\
\omega \geqslant \omega_2, \; \alpha_1(\omega) = 0, \; \alpha_2(\omega) = 1.
\end{gathered} \tag{3.5}
$$

The bunching point of the spectrum

$$
\omega = \omega_1, \; \omega = \omega_2. \tag{3.6}
$$

For the parameters of the orthotropy, let us examine the second relationship from (3.1). The distribution of the natural frequencies for shells with a positive Gaussian curvature starts from a frequency $\omega = \omega_1$

$$
\begin{gathered}
\omega \leqslant \omega_1, \; M(\omega) = 0, \\
\omega_1 \leqslant \omega \leqslant \omega_2, \; \alpha_1(\omega) = 0, \; \alpha_2(\omega) = \xi_1, \\
\omega \geqslant \omega_2, \; \alpha_1(\omega) = 0, \; \alpha_2(\omega) = 1.
\end{gathered} \tag{3.7}
$$

Bunching of natural frequencies with $\omega = \omega_2$. With $\chi < 0$, the spectrum is analogous to the spectrum of the problem whose basic characteristics are given by formulas (3.5), (3.6).

Let now, for the coefficients of the orthotropy, the third relationship from (3.1) be satisfied, and let the curvature of the shell satisfy the condition

$$\chi \geqslant \frac{c_{11}c_{22} - c_{12}^2 - 2c_{12}c_{33}}{2c_{11}c_{33}} = \chi_2, \tag{3.8}$$

where, in principle, three cases are possible: Shells with a curvature satisfying the given condition can be of either positive or negative Gaussian curvature. With $\chi \geqslant 0$, the initial point of the spectrum $\omega = \omega_1$

$$\begin{gathered}
\omega_1 \leqslant \omega \leqslant \omega_2,\ \alpha_1(\omega) = 0,\ \alpha_2(\omega) = \xi_1,\\
\omega_2 \leqslant \omega \leqslant \omega_*,\ \begin{cases}\alpha_1(\omega) = 0, & \alpha_2(\omega) = \xi_1,\\ \alpha_1(\omega) = \xi_2, & \alpha_2(\omega) = 1,\end{cases}\\
\omega \geqslant \omega_*,\ \alpha_1(\omega) = 0,\ \alpha_2(\omega) = 1.
\end{gathered}$$

With $\omega_2 \leqslant \omega \leqslant \omega_*$, the interval of change in the variable ξ consists of two; correspondingly, the integral I is divided into two; in the first, the integration is carried out from 0 to ξ_1 and, in the second, from ξ_2 to 1. With $\omega = \omega_*$ where ω_* is determined by formula (3.4), Eq. (2.5) has a multiple root $\xi_1 = \xi_2$ in the interval. With this value of the frequency, the integral I diverges. In the sense of a principal value, it does not exist. For shells with a negative Gaussian curvature with $\chi < 0$,

$$\begin{gathered}
0 < \omega \leqslant \omega_1,\ \alpha_1(\omega) = \xi_1,\ \alpha_2(\omega) = \xi_2,\\
\omega_1 \leqslant \omega \leqslant \omega_2,\ \alpha_1(\omega) = 0,\ \alpha_2(\omega) = \xi_2,\\
\omega_2 \leqslant \omega \leqslant \omega_*,\ \begin{cases}\alpha_1(\omega) = 0, & \alpha_2(\omega) = \xi_1,\\ \alpha_1(\omega) = \xi_2, & \alpha_2(\omega) = 1,\end{cases}\\
\omega \geqslant \omega_*,\ \alpha_1(\omega) = 0,\ \alpha_2(\omega) = 1.
\end{gathered}$$

The bunching points of the natural frequencies

$$\omega = \omega_1,\ \omega = \omega_*.$$

If the equality is satisfied in (3.8), then $\omega_2 = \omega_*$; if the equality is satisfied in the third relationship from (3.1), then, in (3.8) the right-hand part is equal to 1. We then consider only the following case (compare the limitations (3.8):

$$\chi \leqslant \frac{c_{11}c_{22} - c_{12}^2 - 2c_{12}c_{33}}{2c_{11}c_{33}}.$$

Analogous to the preceding, here there are two possible cases. If $\chi \geqslant 0$, then, with $\omega \leqslant \omega_1$, $M(\omega) = 0$, and, further on, as in (3.7). For shells with a negative Gaussian curvature, formulas (3.5) and (3.6) are valid.

It was postulated above that $c_{22} > c_{11}$. We now assume that $c_{11} > c_{22}$, i.e., for the coefficients of the orthotropy, the relationships (3.2) are possible, which are obtained from the conditions (3.1) if c_{11} and c_{22} change places in them. The results of an analysis, obtained with relationships (3.1), will hold also with relationships (3.2) if, with a curvature $|\chi| \geqslant (c_{22}/c_{11})^{1/2}$, ω_1 and ω_2 change places in all the expressions given above. This is connected with the fact that, with $c_{22} > c_{11}$, it is always true that $\omega_1 < \omega_2$. But, with $c_{22} < c_{11}$ and $|\chi| \geqslant (c_{22}/c_{11})^{1/2}$, the condition $\omega_1 < \omega_2$. If $|\chi| \leqslant (c_{22}/c_{11})^{1/2}$, all the results are carried over without change.

And finally, if $c_{11} = c_{22} = c$, then, in (3.1), (3.2) the equality $c_{33}^* = C_{33}^{**} = (c - C_{12})/2$ is satisfied, i.e., for the coefficients of the orthotropy, instead of three relationships, only two relationships need be considered

TABLE 1

c_{33}	χ		Start of spectrum	Bunching points
$c_{33}\leqslant c^{*}_{33}$	$\chi\geqslant\chi_1$		ω	$\omega_1,\ \omega_2$
	$0\leqslant\chi\leqslant\chi_1$		ω_1	ω_2
	$\chi\leqslant 0$		0	$\omega_1,\ \omega_2$
$c^{*}_{33}\leqslant c_{33}\leqslant c^{**}_{33}$	$\chi\geqslant 0$		ω_1	ω_2
	$\chi<0$		0	$\omega_1,\ \omega_2$
$c_{33}\geqslant c^{**}_{33}$	$\chi\geqslant 0$	$\chi\geqslant\chi_2$	ω_1	ω_*
	$\chi\leqslant 0$		0	$\omega_1,\ \omega_*$
	$\chi\geqslant 0$	$\chi\leqslant\chi_2$	ω_1	ω_2
	$\chi\leqslant 0$		0	$\omega_1,\ \omega_2$

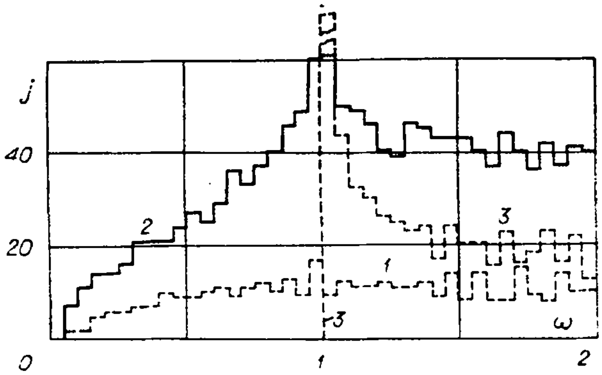

Fig. 4

$$c_{33}\leqslant c,\ c_{33}\geqslant c.$$

For the frequencies ω_1 and ω_2 we have $\omega_1\leqslant\omega_2$. The equality here is satisfied only for shells with a curvature $|\chi| = 1$, in particular, for a spherical panel.

4. Relationships (3.1) between the parameters of the orthotropy in terms of technical constants can be written in the form

$$G\leqslant E_1/2(E_1/E_2+\nu_1),\ E_1/2(E_1/E_2+\nu_1)\leqslant G\leqslant E_1/2(1+\nu_1),\ G\geqslant E_1/2(1+\nu_1). \tag{4.1}$$

Relationships (3.2) are written analogously; in writing (4.1) it is only necessary to exchange the places of the subscripts 1 and 2. Article [7] gives materials whose characteristics satisfy similar relationships. It also gives the literature references from which these data were taken. Relationships of the type of (3.1) and (3.2) do not go beyond the limitations imposed on the coefficients of the orthotropy in the theory of elasticity.

The parameters of the orthotropy of the material and the geometry of shells can, in principle, be interconnected in another form, e.g.,

$$c_{33}\leqslant c_{\chi},\quad c_{\chi}\leqslant c_{33}\leqslant c_{\chi\chi},\quad c_{33}\geqslant c_{\chi\chi},$$

$$c_{\chi}=\frac{c_{11}c_{22}-c_{12}^2}{2(c_{22}+c_{12}\chi)}\chi,\quad c_{\chi\chi}=\frac{c_{11}c_{22}-c_{12}^2}{2(c_{22}\chi+c_{12})}\quad(\chi\geqslant 0,\ c_{22}\geqslant c_{11}).$$

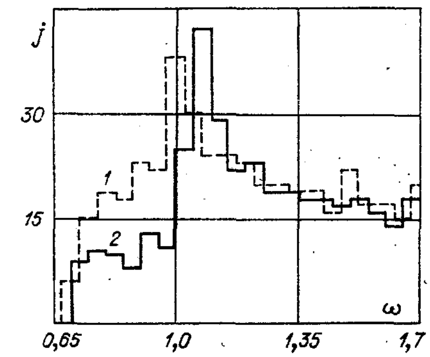

Fig. 5

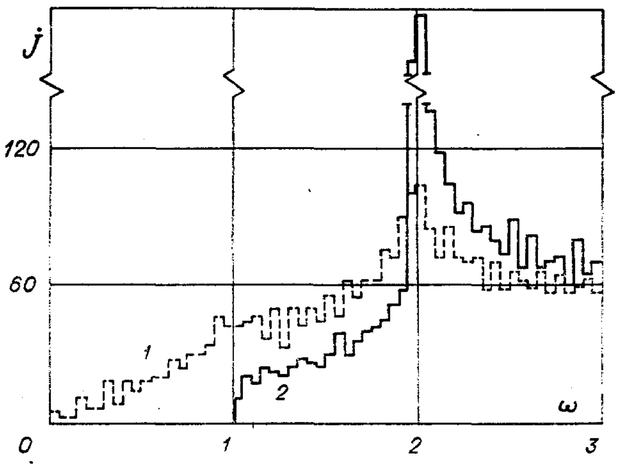

Fig. 6

Each relationship corresponds to its own type of spectrum. The results obtained characterize the effect of the coefficients of the system (1.1) on the spectrum of the problem. Table 1 gives asymptotic expressions for the initial points and the bunching points of the spectrum for the case $c_{22} > c_{11}$. If $c_{22} < c_{11}$, c_{33}^* and c_{33}^{**} must change place in Table 1. ω_1 and ω_2 also change place with $|\chi| > (c_{22}/c_{11})^{1/2}$. Figures 1-3 give the asymptotics of all the types of spectra characteristic for orthotropic shells. The curves shown by the solid lines for $M(\omega)$ are characteristic also for isotropic shells, the dashed lines are characteristic only for orthotropic shells; the notation is the same in Table 1 and Figs. 1-3. Using the table, it is easy to determine which spectrum corresponds to which relationship between the parameters of the orthotropy and the geometry. The frequencies were calculated using formula (1.2); they were then grouped into intervals with a spacing of 0.05. All the calculations were made with the following values of the parameters of the orthotropy: $c_{11} = 1$, $c_{22} = 0.25$, $c_{12} = 0.1$, and with three values of $c_{33} = 0.05, 0.2, 0.5$. In this case, one of the conditions (3.1) was satisfied consecutively. Here, the parameters of the orthotropy are referred to a maximal value, correspondingly, of the frequency in Figs. 4-6 $\omega = \omega c_{11}^{-1/2}$. In Fig. 4, curves 1, 2 characterize the distribution of the natural frequencies of a cylindrical shell with a ratio $l/R = 2$ and $c_{33} = 0.2$; the lower curve corresponds to a shell with $R/h = 100$, the upper with $R/h = 400$; curve 3 corresponds to a shell with a curvature $\chi = 1/2$, $a/b = 1$, i.e., with our parameters of the orthotropy; the bunching point $\omega_1 = \omega_2 = 1$ coincides with the start of the spectrum. This can be seen well on the curve, where the maximum is equal to $j = 96$. The calculations were made with $c_{33} = 0.2$. Figure 5 gives the distributions of the frequencies of shells with the following parameters: $\chi = 1/3$, $R/h = 1600$; $a/b = 1$, curve 1 with $c_{33} = 0.2$; curve 2 with $c_{33} = 0.5$; the point of bunching is well expressed with $\omega = 1$. The density of the natural frequencies for shells with a negative Gaussian curvature is shown in Fig. 6 by the curve 1, where $\chi = -1$, $R/h = 6400$; $a/b = 1$, $c_{33} = 0.5$; here there is also given the distribution of the frequencies for a spherical panel ($\chi = 1$, curve 2) with the parameters $R/h = 6400$, $a/b = 1$, $c_{33} = 0.5$.

Formula (2.4) gives the asymptotic distribution of the density of the natural frequencies. The results of a numerical calculation of the initial part of the spectrum using formula (1.2) coincide with the conclusions drawn with an analysis of the asymptotic density. By varying the geometry of the shell and the mechanical properties of the material, it is possible to control the spectrum, shifting either the start of the spectrum or the bunching points out of the undesirable region. In this case, there is a decrease in the possibility of resonance phenomena.

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